Jour. Ind. Soc. Ag. Statistics
Vol. XXXIX, No. 2 (1987), pp. 148-153

A NON-PARAMETRIC APPROACH TO ANOVA

D. V. SUBBA RAO and M. SUBRAMANYAM
C.T.R.I., Rajahmundry

(Received : September, 1985)

SUMMARY

A non-parametric approach as an alternative to analysis of variance and the usual *F*-test in the RBD experiments is suggested using the method of paired comparison. This method is illustrated and shown to give the same result as of ANOVA, besides giving information on the classification of varieties into three distinct groups viz., superior performers, medium performers and low performers.

Keywords : Ranks; Paired Comparison; Heterogeniety χ^2; Grouping treatments.

Introduction

Often, a plant breeder comes across the problem of choosing the superior varieties (treatments) or ranking them in a plant breeding trial. He naturally intends to classify varieties into superior, inferior and average performers. The usual way is, to conduct a field experiment with a randomised block design or some other advanced design and adopt analysis of variance technique and rank the varieties on the basis of their average performance over all replications provided these varieties turn out to be significant. If they do not turn out to be significant he will be in a fix to classify them. Further it is observed, many a time, the experimental data do not satisfy the assumption underlying the ANOVA and has to be suitably transformed. To overcome these problems a rigid classification of the varieties into significantly superior performers, non-significant average performers and significantly low performers is suggested in the present study, using the method of paired comparison to a data emanated from an experiment with a randomised block design. Bradley and Terry [1], Starks and David [2], Sadasivan and Rai [3], and Gupta and Rai [4] suggested the use of method of paired comparisons in-case-of qualitative characters like taste and flavour where no meaningful absolute measurements are possible. The method of Starks and David [2] has also been reviewed by converting the quantitative data into qualitative data which could be considered as an alternative to ANOVA.

2. Methodology

The performance of two treatments are generally assessed by ranking them as 1 or 2 by a judge who will score the treatments on the basis of their superiority in taste, flavour etc., in the method of paired comparisons. The material will be tested in pairs by a single judge for all possible pairs of a set of treatments. This testing will be carried out by 'r' independent judges to provide replications. The number of times a particular treatment ranked above a given set of treatments is considered as its superiority over others. Data from a randomised block experiment is assumed to have emanated from an experimental material under identical conditions. Hence the numerical differences in their responses are attributed to the performance of the treatments. In this study, the numerical difference between a pair of treatments greater than zero will receive a rank 1 or 0 otherwise. The ranking as described above will be carried out separately in each replicate for all the t_{c_2} pairs of treatments and the number of times, the ith treatment ranked above the other set of treatments over all replications could be tabulated as in Table 1.

TABLE 1—DISTRIBUTION OF RANK SUMS

Treatments	*Number of times ith treatment received rank 1*	*Number of times ith treatment failed to receive rank 1*	*Total*
1	r_{11}	r_{01}	T_1
2	r_{12}	r_{02}	T_2
3	r_{13}	r_{03}	T_3
.	.	.	.
.	.	.	.
i	r_{1i}	r_{0i}	T_i
.	.	.	.
.	.	.	.
t	r_{1t}	r_{0t}	T_t
Total	C_1	C_2	G

Here r_{1i} denote the number of times ith treatment received rank 1 and r_{0i} denote the number of times ith treatment failed to receive rank 1. Let $T_1, T_2, \ldots T_i \ldots T_t (= T)$ denote the row totals and C_1 and C_2 be the column totals and G be the grand total. Then it could be shown that

$$r_{1i} + r_{0i} = r(t-1) = T_i \quad (i = 1, 2 \ldots . t)$$
$$G = rt(t-1)$$

and $$C_1 = C_2 = \frac{rt(t-1)}{2} \tag{2.1}$$

If there is no real difference between the ith treatment and other treatments then r_{1i} and r_{0i} will be equally distributed and will result in a value of $\chi^2 = 0$. In other cases, it gives us a value for χ^2 with 1 d.f. which can be tested for its significance at 5% or 1% tabulated values of χ^2 distribution viz., 3.84 or 6.63. The value of χ^2 for classes r_{1i} and r_{0i} are equal and over both the classes for an ith treatment can be worked out from the reduced formula

$$\chi_i^2 = \frac{[2r_{1i} - r(t-1)]^2}{r(t-1)} \tag{2.2}$$

Thus the assumption of equality of distribution of the ranks r_{1i}, r_{0i} and the significance of χ_i^2 will enable us to group the treatments into three broad groups; (i) Non-significant χ_i^2 with average ranks of r_{1i}; (ii) Highly significant χ_i^2 with high ranks of r_{1i} and (iii) Highly significant χ_i^2 with low ranks of r_{1i}. High values of r_{1i} indicate high performers, low values of r_{1i} indicate low performers in the experiment. Values of r_{1i} around the average value $= r(t-1)/2$ are medium performers which never turn out to be significant. Heterogeneity χ^2 could be worked out by pooling individual χ_i^2 values to test the over all performance of the set of treatments.

$$H = \text{Heterogeneity } \chi^2 = \frac{\sum_{i=1}^{t} [2r_{1i} - r(t-1)]^2}{r(t-1)}$$
$$= \sum_{i=1}^{t} \chi_i^2 = 1 \tag{2.3}$$

Now H will be tested against χ^2 at 5% and 1% levels of significance with t d.f. This grouping of treatments appeared to be more useful in screening of varieties etc. and is appropriate than the existing analysis of variance and C.D. values avoiding confusion in the bar method of grouping.

Starks and David [2] evolved a D statistic for testing the differences among the sums of ranks given by

$$D = \left[4 \sum_{i=1}^{n} a_i^2 - 1/4\, t n^2 (t-1)^2 \right] \Big/ nt \tag{2.4}$$

D is distributed as χ^2 with $(t-1)$ d.f. for which C.D.'s at 5% and 1% levels of significance are given by

$$\begin{aligned} \text{C.D. at } 5\% &= 1.96\,(1/2\; nt)^{1/2} + 1/2 \\ \text{C.D. at } 1\% &= 2.5758\,(1/2\; nt)^{1/2} + 1/2 \end{aligned} \tag{2.5}$$

where a_i is the number of times ith treatment ranked above the other treatments with n replications and t treatments. The D Statistic considered above is an alternative to 'F' test in the analysis of variance to test the significance of the treatment differences when the responses are not measurable and usual assumptions of analysis of variance do not hold good. The H Statistic viz., Heterogeneity χ^2 as given at (2.3) corresponds to D Statistic given at (2.4) for testing the significance of treatments using rank sums. However individual treatment comparisons using rank sums for detecting significant differences could be made using (2.2) viz. individual χ^2 values with 1 d.f. Further this makes it possible to rigidly classify the set of treatments that are highly promising, medium and low performers which could not be done by the earlier procedures.

3. Example

To illustrate the methodology developed in section 2, data from a field experiment involving the screening of 12 flue-cured virginia tobacco varieties conducted in four replications at research farm, Kateru of the Central Tobacco Research Institute during 1982-83 crop season, have been utilised. In Table 2 the rank sums r_{1i} and r_{0i} for each of the varieties along with the corresponding mean yields of cured-leaf in Kg/ha and χ_i^2 values are presented.

For testing the over all differences among the treatments, the Statistics pertaining to the three methods F, D, and H are presented in Table 3 along with the theoretical values of F with 11, 33 d.f. and χ^2 with 12 d.f. at 5% and 1% levels of significance. C.D. values for F and D methods for testing the differences between pairs of treatments are also given in Table 3.

It could be seen from the above table, differences among the treatments are found to be highly significant in all the three methods. The relative position of the treatments in Table 2 remained the same for the values

TABLE 2—RANK SUM AND MEAN YIELD FOR TREATMENTS

Treatments	*Some of ranks for* r_{1i}	r_{0i}	*Mean yield in* Kg/ha	χ_i^2 (1 d.f.)
A	26	18	1490.45	1.4546
B	27	17	1548.61	2.2728
C	20	24	1471.35	0.3636
D	5	39	1299.48	26.2728**
E	35	9	1739.58	15.3636**
F	28	16	1592.88	3.2728
G	6	38	1074.65	23.2728**
H	17	27	1411.46	2.2728
I	26	18	1533.85	1.4546
J	10	34	1342.01	13.0910**
K	22	22	1457.29	0
L	42	2	1901.91	36.3636**

* = Significant at 5%
** = Highly Significant at 1%
χ^2 5% for 1 d.f. = 3.84
χ^2 1% for 1 d.f. = 6.63

TABLE 3—COMPARISON OF *F*, *D* AND *H* STATISTICS

	F *ANOVA Method*	*D* *Stark-David Method*	*H* *Heterogeneity* χ^2
Calculated values	5.06**	115**	125.4545**
Theoretical at 5%	2.03	19.68	21.03
Theoretical at 1%	2.724	24.73	26.22
S.E. of the difference	132.11	4.899	—
C.D. at 5%	258.93	10.102	—
C.D. at 1%	340.31	13.12	—

* Significant at 5%
** Highly Significant at 1%

of r_{1i} as well as actual treatment means. However an additional information for grouping the varieties into superior performers, medium performers and inferior performers could be given by H-Statistic as shown in Table 4.

TABLE 4—CLASSIFICATION OF VARIETIES

Performance	*Significant* χ^2 *with High performance*	*Non significant* χ^2 *with medium performance*	*Significant* χ^2 *with low performance*
Varieties	*L, E*	*B, A, C, F, H, I, K*	*J, G, D*
Average yield of the group in Kg/ha	1820.27	1500.84	1238.71
Average Rank sum of the group	38.75	19.71	7.00

ACKNOWLEDGEMENTS

The authors are thankful to the Director, Central Tobacco Research Institute, Rajahmundry for encouragement throughout the course of investigation. They are also thankful to the referees for their valuable comments.

REFERENCES

[1] Bradley, R. A., Terry, M. R. (1952) : Rank analysis of incomplete block designs-I : The method of paired comparisons, *Biometrika*, **39** : 324-45.

[2] Starks, T. H. and David, H. A. (1961) : Significance tests for paired comparisons experiments, *Biometrika*, **48** : 95-108.

[3] Sadasivan, G. and Rai, S. C. (1973) : A Bradley-Terry Model for standard comparison pairs, *Sankhya*, B **35** : 25-34.

[4] Gupta, S. C. and Rai, S. C. (1980) : Rank analysis in paired comparisons, *J. Ind. Soc. Agric. Stat.*, **32** : 87-98.